Sonic Boom Waveforms and Amplitudes in a Real Atmosphere

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A simple method for determining sonic boom waveshapes and amplitudes in a still stratified atmosphere is presented. The effects of acoustic impedance, ray tube area, and nonlinear wave steepening and shock formation are each treated separately. By restricting consideration to the waves directly below an aircraft in steady flight, it is possible to present all the information needed to calculate wave shapes and amplitudes without use of a computer. The results for the U.S. Standard Atmosphere are presented and compared to those calculated for various model atmospheres and to the results of various approximations.

Nomenclature

a = speed of sound
A = area of ray tube
C_A = ray tube area correction [see Eq. (7)]
C_I = acoustic impedance correction [see Eq. (5)]
C_T = nonlinear advance coefficient [see Eq. (2)]
H = density scale height of isothermal atmosphere
K = N wave atmospheric correction [see Eq. (11)]
M = $U/a(r)$ = Mach number of aircraft relative to speed of sound at any given altitude
p = pressure
δp = perturbation pressure of wave relative to local p
Δp = pressure jump across initial shock of N wave
r = radius from flight path
s = distance along ray tube
t = time
u = fluid velocity in wave propagation direction
U = aircraft velocity
x = axial coordinate
y = spanwise coordinate
z = vertical coordinate
γ = ratio of specific heats
θ = azimuthal angle

Subscripts

h = reference altitude where initial δp curve is given
i = isothermal atmospheres
u = uniform atmospheres

Introduction

METHODS of calculating sonic boom waveforms and amplitudes in a uniform atmosphere have been well known for some time.[1–5] However, the modifications to account for the effects of a nonuniform stratified atmosphere have only recently been treated completely by Hayes.[6] Various parts of the problem have been dealt with in Refs. 7–11 and a review of the simplest theory has been given recently by Seebass.[12] Hayes outlined a general approach including arbitrary flight paths in a general atmosphere including steady winds. A computer program applying these methods to any layered atmosphere has been developed by Hayes, Haefeli, and Kulsrud.[13]

The present paper treats the case of steady flight and no winds, making it possible to present all the information necessary to find sonic boom waveforms and amplitudes directly below the aircraft flight track without the use of a computer. The effects of acoustic impedance, ray tube area change, and nonlinear wave steepening and shock formation are presented individually and the results for an N wave compared to some early calculations for this special case.

Sonic boom waves are typically weak in the aerodynamic sense with perturbation pressures of order 10^{-2} of ambient pressure a few body lengths from the aircraft and of order 10^{-3} of ambient pressure at the ground. In addition, the typical wavelengths of the boom are small compared to the scale of variations in properties of the atmosphere, and thus, the methods of geometrical acoustics may be applied with corrections for nonlinear wave steepening effects made following Landau[1] or Whitham.[2] First, it is shown that the amplitude of an initial pressure wave form is scaled by factors C_I and C_A accounting for the acoustic impedance and ray tube area changes during the waves propagation in the atmosphere. This gives

$$\delta p = C_I C_A \delta p_h \tag{1}$$

Next it is shown that the cumulative nonlinear effects on the wave shape are accounted for by adjusting the time of arrival of each part of the wave by

$$\Delta t = C_T(\delta p/p)_h r_h^{1/2} \tag{2}$$

Finally shock waves are inserted, making the δp curve single valued. In the case of an N wave, the shocks can be found analytically and the results given in terms of a factor K where the initial shock strength of the N wave is given by

$$\Delta p = K(p/p_h)^{1/2}\Delta p_u \tag{3}$$

Analysis

In a quiescent medium, the acoustic approximation treats any portion of a given wave as a one-dimensional wave propagating at the local speed of sound in a ray tube defined by normals to the wave front (location of constant phase) (see Fig. 1). The ray tubes may be constructed from Fermat's principle or Snell's law. Then the strength of the pressure perturbation is determined from

$$\delta p(A/\rho a)^{1/2} = B \tag{4}$$

where B is an energy invariant and is constant along the ray tube.[14] The quantity ρa is sometimes called the acoustic impedance and is easily tabulated for any given atmosphere. The factor

$$C_I = (\rho a/\rho_h a_h)^{1/2} \tag{5}$$

is plotted for sea level in the U.S. Standard Atmosphere[15] in Fig. 2. Also shown is $(p/p_h)^{1/2}$, which has been used in the past to approximate this effect. It is seen to be a good ap-

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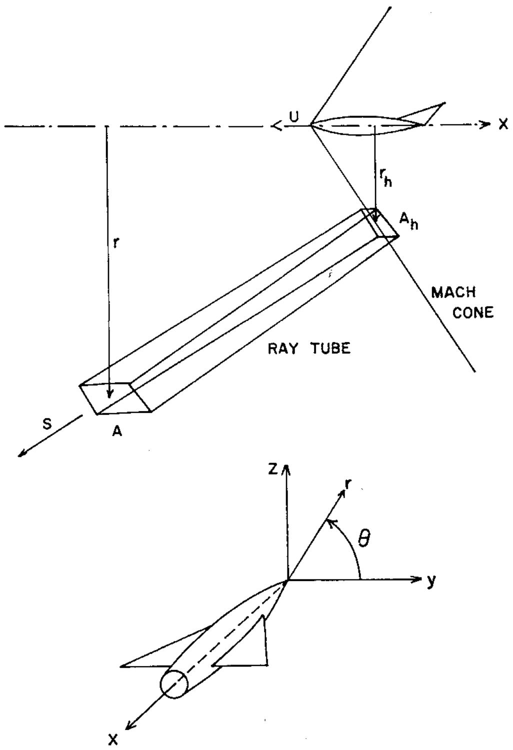

Fig. 1 Coordinate systems and notation.

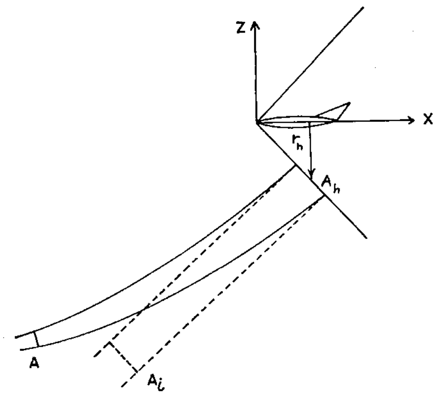

Fig. 3a ***x*-*y* plane ray curvature.**

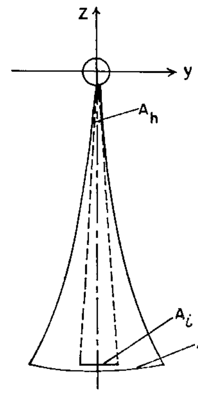

Fig. 3b ***y*-*z* plane ray curvature.**

proximation but, on the other hand, the exact C_I is no harder to evaluate.

For the present case, the ray tube areas a were found by applying Snell's law to rays originating normal to the Mach cone at $\theta \cong -\pi/2$. The sound speed gradient bends the rays forward in the $x = z$ plane decreasing the distance between rays, particularly for rays approaching glancing incidence (see Fig. 3a). This effect is most important for low Mach numbers where rays originate closer to glancing incidence. In the y-z plane, the increasing sound speed increases the distance between rays slightly even for the rays directly below the aircraft as shown in Fig. 3b. Using Snell's law, the expression for the area was found to be

$$A = \frac{A_h M_h}{r_h}\left[1 - \frac{1}{M(r)^2}\right]^{1/2} \int_0^r \frac{dr}{[M(r)^2 - 1]^{1/2}} \tag{6}$$

where $M(r) = U/a(r)$. This agrees with the area found by Randall using Fermat's principle.[8] C_A is given by

$$C_A = \left(\frac{A}{A_h}\right)^{-1/2} \tag{7}$$

and $C_A(r/r_h)^{1/2}$ is the factor accounting for the effect of the actual ray tube area compared to that in a constant speed of sound atmosphere where $A = A_h r/r_h$. $C_A(r/r_h)^{1/2}$ is plotted in Fig. 4 for sea level in the U.S. Standard Atmosphere. The large values of $C_A(r/r_h)^{1/2}$ at low Mach number are due to the dominance of the effect shown in Fig. 3a, whereas at higher Mach numbers, the smaller lateral spreading effect of Fig. 3b is dominant. At very large altitudes, the effect of the slight increase in the speed of sound above 20 km is evident.

Next, the nonlinear effects on the wave propagation are accounted for. The propagation speed of any portion of the wave is $u + a$ where u and a are the complete local values of the fluid velocity and sound speed. By accounting for only the first-order effects of δp on u and a, the propagation time per unit distance can be written $dt/ds = (1/a) + (\gamma + 1)/(2\gamma a)\delta p/p$. Using Eq. (4) and integrating along a ray from s_h to s, the time of arrival of a given part of the wave can be

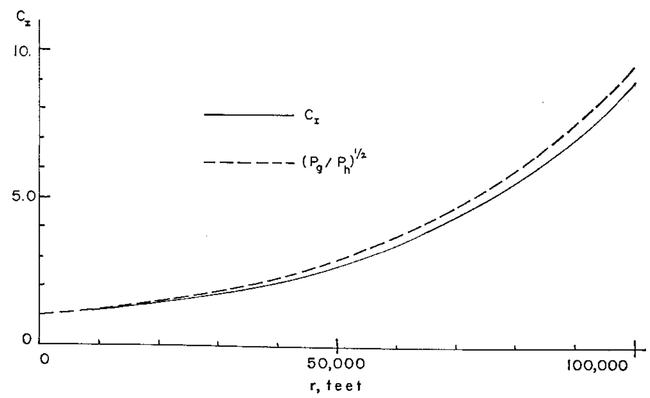

Fig. 2 Acoustic impedance correction factor and $(p/p_h)^{1/2}$ vs height.

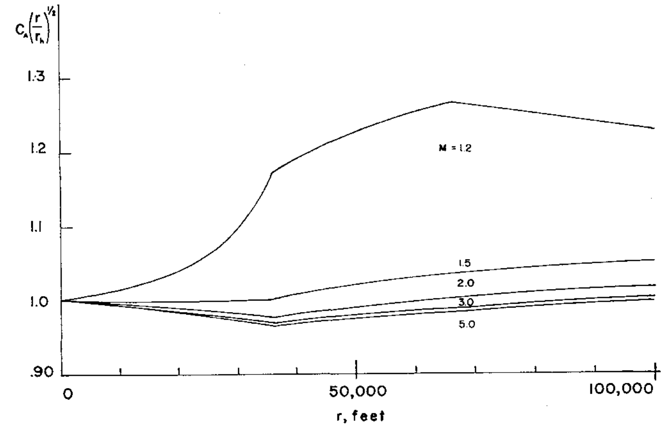

Fig. 4 Ray tube area correction factor vs height.

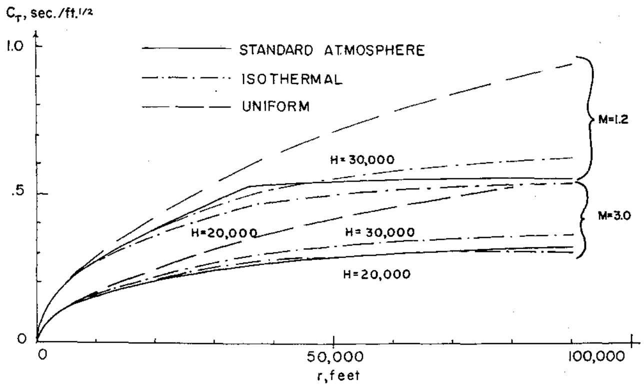

Fig. 5 Advance factor for various atmospheres.

written

$$t = \int_{S_h}^{S} \frac{ds}{a} - \frac{\gamma + 1}{2\gamma a_h}\left(\frac{\delta p}{p}\right)_h \int_{S_h}^{S} \frac{a_h p_h}{ap}\left(\frac{\rho a A_h}{\rho_h a_h A}\right)^{1/2} ds$$

Thus, the advance in time of the arrival of a portion of the wave compared to an infinitesimal acoustic wave is

$$\Delta t = \left(\frac{\delta p}{p}\right)_h \frac{\gamma + 1}{2\gamma a_h} \int_{S_h}^{S} \frac{p_h}{p}\left(\frac{\rho a_h A_h}{\rho_h a A}\right)^{1/2} ds$$

Transforming from s to r with $ds/dr = M/\beta$ where $M = U/a(r)$ and taking $r_h \ll r$, we define‡

$$C_T = \frac{\Delta t}{(\delta p/p)_h r_h^{1/2}} = \frac{\gamma + 1}{2\gamma a_h} \int_0^r \frac{p_h}{p}\left(\frac{\rho a_h}{\rho_h a}\right)^{1/2}\left(\frac{A_h}{r_h A}\right)^{1/2} \frac{M}{\beta} dr \qquad (8)$$

C_T has been computed for the U.S. Standard Atmosphere and examples are plotted in Fig. 5, along with C_T for a constant-property atmosphere

$$C_{T_u} = \frac{\gamma + 1}{\gamma a_h} \frac{M}{\beta} r^{1/2} \qquad (9)$$

and for an isothermal atmosphere with density scale height H

$$C_{Ti} = \frac{\pi^{1/2}}{2} \frac{\gamma + 1}{\gamma a_h} \frac{M}{\beta} (2H)^{1/2} \operatorname{erf}\left(\frac{r}{2H}\right)^{1/2} \qquad (10)$$

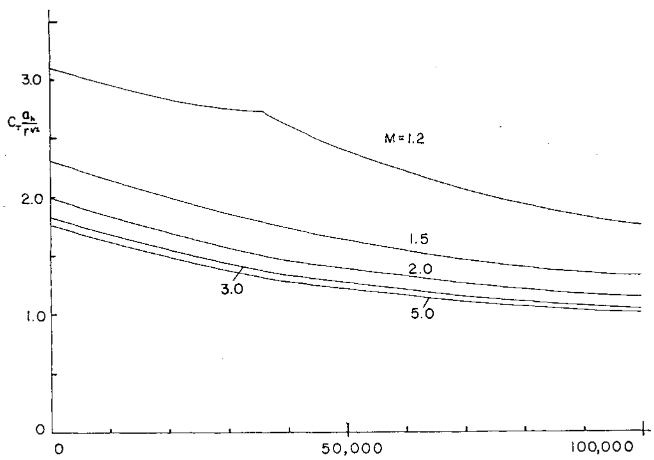

Fig. 6 Nondimensional advance factor for standard atmosphere.

‡ $(\delta p/p)_h r_h^{1/2}$ is related to the Whitham F function by $(\delta p/p)_h r_h^{1/2} = (2\beta)^{-1/2}\gamma M_h^2 F$. The time variable t is related to the axial variable x by $x = a_h M_h t$.

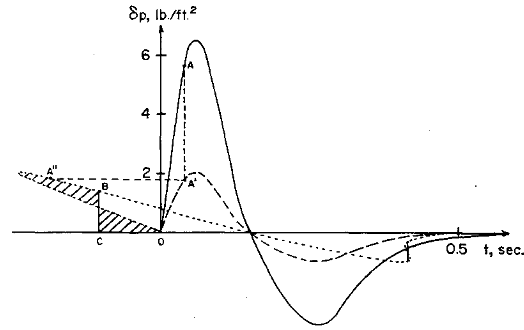

Fig. 7 Example of application.

In the nonuniform atmospheres, it can be seen that the advance approaches a finite limit and is always less than it would be in the uniform atmosphere. Thus, the possibilities for finite rise time aircraft and midfield signatures are somewhat improved when considering the real atmosphere as first pointed out by Hayes. It can be seen that the isothermal atmosphere with a suitably chosen scale height of about 20,000 ft gives a reasonable approximation to C_T for the U.S. Standard Atmosphere. The nondimensional parameter $C_T a_h / r^{1/2}$ is plotted in Fig. 6 for the U.S. Standard Atmosphere and a full range of Mach numbers.

Next, an example of the use of these results is given. An idealization of a curve obtained by standard area rule methods[16] is shown as the solid line in Fig. 7 for $r = 60{,}000$ and r_h 500 ft. Figures 2 and 5 give C_I and C_A as 3.48 and 0.0895, respectively. For $M = 3.0$, multiplication by $C_A C_I = 0.313$ moves a typical point from A to A' resulting in the dashed curve. Then, each point on the curve is advanced in time by $C_T(\delta p/\rho)_h r_h^{1/2}$ resulting in the dotted curve by moving points from A' to A'', for example. Finally, as shown by Landau and Whitham, shocks are inserted as vertical lines which make the dotted δp curve single valued while keeping the area under the curve constant. A shock is shown in the figure as the line BC and the areas balanced by this shock are shaded.§

Asymptotically far from an aircraft, the waveform often tends to an N wave.[1,2] Applying the preceding theory and determining the shock analytically, the amplitude of the pressure jump Δp across the leading shock of an N wave can be expressed $\Delta p = (p/p_h)^{1/2} K \Delta p_u$ where Δp_u is the well-known

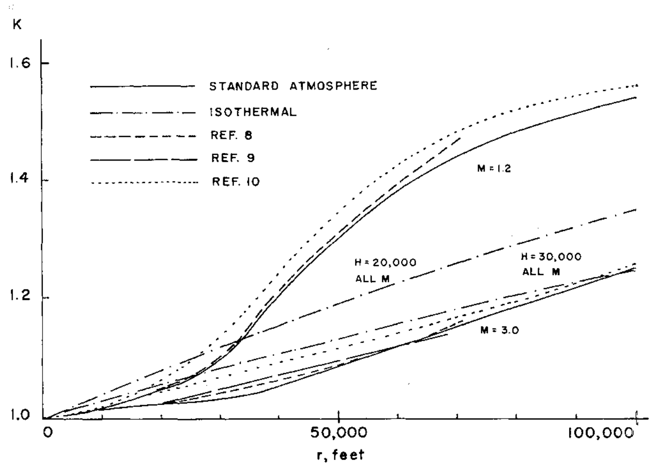

Fig. 8 N wave correction factor for various atmospheres.

§ As an alternate to shifting the δp curve by Δt and using vertical lines to patch in the shocks, one can use a procedure analogous to Whitham's[2] and locate the shocks with lines of slope $d\delta p/dt = C_A C I p_h / C_T r_h^{1/2}$ balancing areas on the δp curve or $d\delta p_h/dt = p_h / C_T r_h^{1/2}$ balancing areas on the δp_h curve.

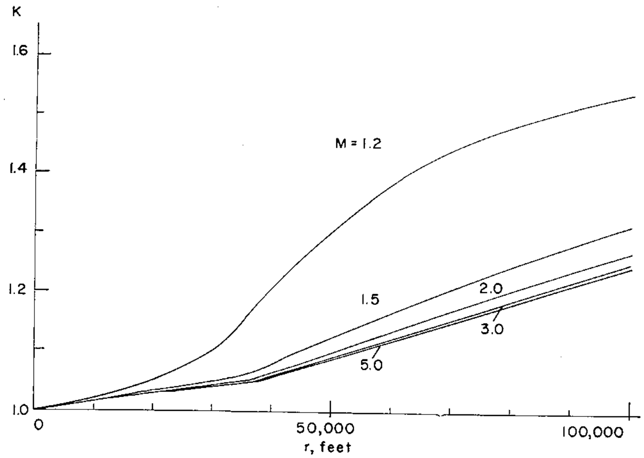

Fig. 9 *N* wave correction factor for standard atmosphere.

result for a uniform atmosphere (Ref. 3 for example) and

$$K = \left(\frac{p_h}{p}\right)^{1/2} C_I C_A \left(\frac{r}{r_h}\right)^{1/2} \left[\frac{1}{2r^{1/2}} \int_0^r \times \frac{p_h}{p}\left(\frac{\rho a_h}{\rho_h a}\right)^{1/2} \frac{M\beta_h}{M_h \beta}\left(\frac{A_h r}{A r_h}\right)^{1/2} \frac{dr}{r^{1/2}}\right]^{-1/2}$$

which can be expressed

$$K = \left(\frac{p_h}{p}\right)^{1/2} C_I C_A \left(\frac{r}{r_h}\right)^{1/2} \left[\frac{\gamma}{\gamma + 1} \frac{\beta_h}{M_h} \frac{C_T a_h}{r^{1/2}}\right]^{-1/2}$$

Examples of the factor K are plotted in Fig. 8.[17] The amplitude of Δp is greater than that in the uniform atmosphere primarily because Δt approaches a finite limit in the real atmosphere as shown in Fig. 6. This limits the part of the δp curve engulfed by the shock. Some of the earlier N wave results of Refs. 8–10 are shown for comparison. The present N wave result is the same as Randall's[8] and the calculations agree with his results if the same model atmosphere is used. Reference 8 uses an approximate approach and Ref. 10 apparently employed an erroneous ray tube area expression. K's for a full range of Mach numbers in the U.S. Standard Atmosphere are plotted in Fig. 9.

Conclusion

A simple method for finding sonic boom waveforms and amplitudes below an aircraft flying in a stratified still atmosphere has been presented. The coefficients for the U.S. Standard Atmosphere were given and compared to those for other ideal atmospheres. For Mach numbers of 2.0 or greater, a suitably chosen isothermal atmosphere is a reasonable approximation to the real atmosphere.

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